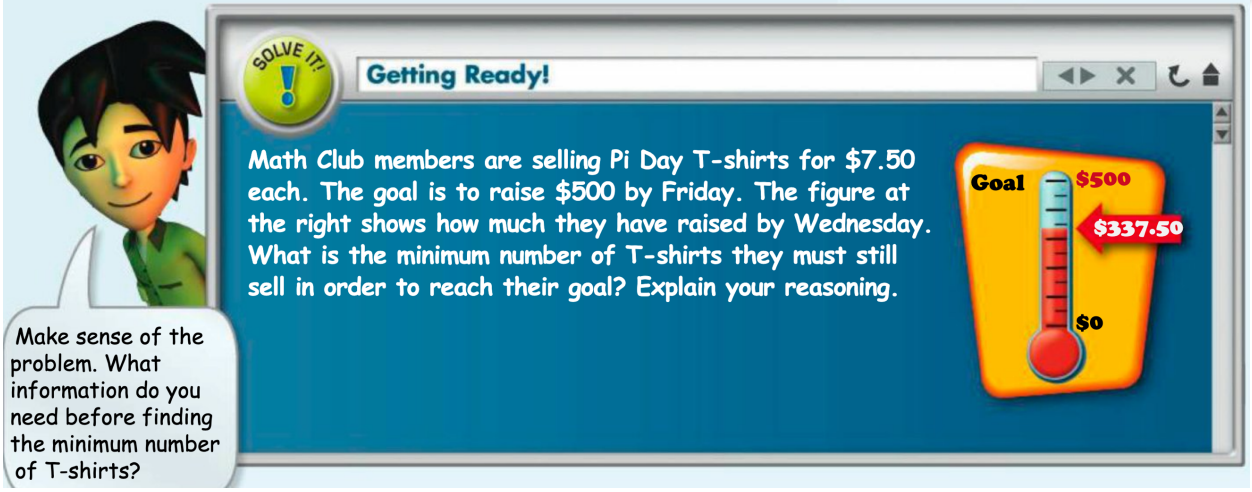
3-4 Solving Multi-Step Inequalities



Let t = t-shirts

$7.5t + 337.50 \geq 500$

$-337.50 \quad -337.50$

$\frac{7.5t}{7.5} \geq \frac{162.5}{7.5}$

$t \geq 21.6$

22 t-shirts

You solve a multi-step inequality in the same way you solve an equation. You use the properties of inequality to transform the original inequality into a series of simpler, equivalent inequalities.

The properties of inequality are very similar to the properties of equality except for this important difference:

When you multiply or divide both sides of an inequality by a negative, you must reverse the inequality symbol

take note

Key Concept Multiplication Property of Inequality

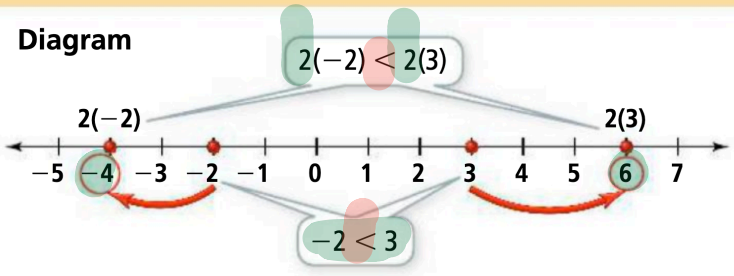
Words

Let a, b, and c be real numbers with $c > 0$.

If $a > b$, then $ac > bc$.

If $a < b$, then $ac < bc$.

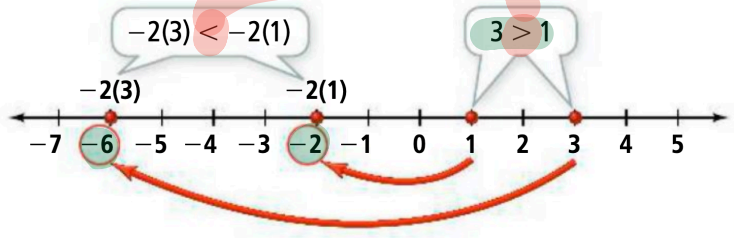
Diagram



Let a, b, and c be real numbers with $c < 0$.

If $a > b$, then $ac < bc$.

If $a < b$, then $ac > bc$.



These properties are also true for inequalities using $\geq$ and $\leq$.

(The Division Property of Inequality is similar to the Multiplication Property of Inequality above)

PROBLEM 1: USING MORE THAN ONE STEP

Solve each inequality. Graph your solutions.

a) $-6a - 7 \leq 17$

$+7 \quad +7$

$\frac{-6a}{-6} \leq \frac{24}{-6}$

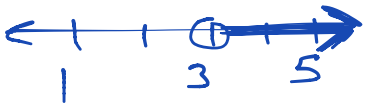
$a \geq -4$

b) $9 + 4t > 21$

$-9 \quad -9$

$\frac{4t}{4} > \frac{12}{4}$

$t > 3$



c) $-4 < 5 - 3n$

$-5 \quad -5$

$\frac{-9}{-3} < \frac{-3n}{-3}$

$3 > n$ or $n < 3$

d) $-12 + 6r < 9$

$+12 \quad +12$

$\frac{6r}{6} < \frac{21}{6}$

$r < \frac{7}{2}$

$r < 3.5$

e) $6 - 3p \geq -9$

$-6 \quad -6$

$\frac{-3p}{-3} \geq \frac{-15}{-3}$

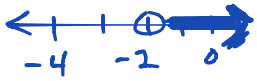
$p \leq 5$

f) $-5y - 2 < 8$

$+2 \quad +2$

$\frac{-5y}{-5} < \frac{10}{-5}$

$y > -2$



PROBLEM 3: USING THE DISTRIBUTIVE PROPERTY

Solve each inequality. Check your solutions.

a) $3(t + 1) - 4t \geq -5$

$3t + 3 - 4t \geq -5$

$-t + 3 \geq -5$

$-3 \quad -3$

$\frac{-1t}{-1} \geq \frac{-8}{-1}$

$t \leq 8$ ✓

b) $15 < 5 - 2(4m + 7)$

$15 < 5 - 8m - 14$

$15 < -9 - 8m$

$+9 \quad +9$

$\frac{24}{-8} < \frac{-8m}{-8}$

$-3 > m$ or $m < -3$

c) $-(7c - 18) - 2c > 0$

$-7c + 18 - 2c > 0$

$-9c + 18 > 0$

$-18 \quad -18$

$\frac{-9c}{-9} > \frac{-18}{-9}$

$c < 2$

d) $4(6y - 12) - 2y \geq -4$

$24y - 48 - 2y \geq -4$

$22y - 48 \geq -4$

$+48 \quad +48$

$\frac{22y}{22} \geq \frac{44}{22}$

$y \geq 2$

e) $-4(d + 5) - 3d > 8$

$-4d - 20 - 3d > 8$

$-7d - 20 > 8$

$+20 \quad +20$

$\frac{-7d}{-7} > \frac{28}{-7}$

$d < -4$

f) $30 > -2(5x + 15) - 12x$

$30 > -10x - 30 - 12x$

$30 > -22x - 30$

$+30 \quad +30$

$\frac{60}{-22} > \frac{-22x}{-22}$

$-\frac{30}{11} < x$

PROBLEM 4: SOLVING AN INEQUALITY WITH VARIABLES ON BOTH SIDES

Solve each inequality.

a) $6n - 1 > 3n + 8$

$-3n \quad -3n$

$3n - 1 > 8$

$+1 \quad +1$

$\frac{3n}{3} > \frac{9}{3}$

$\boxed{n > 3}$

b) $3b + 12 > 27 - 2b$

$+2b \quad +2b$

$5b + 12 > 27$

$-12 \quad -12$

$\frac{5b}{5} > \frac{15}{5}$

$\boxed{b > 3}$

c) $4t + 17 > 7 + 5t$

$-4t \quad -4t$

$17 > 7 + t$

$-7 \quad -7$

$\boxed{10 > t}$

or

$t < 10$

d) $5f + 8 \geq 2 + 6f$

$-5f \quad -5f$

$8 \geq 2 + f$

$-2 \quad -2$

$\boxed{6 \geq f}$

or

$f \leq 6$

f) $3(3m - 4) \leq -2(6m - 6)$

$9m - 12 \leq -12m + 12$

$+12m \quad +12m$

$21m - 12 \leq 12$

$+12 \quad +12$

$\frac{21m}{21} \leq \frac{24}{21}$

$\boxed{m \leq \frac{8}{7}}$

g) $6 - 3p > 4 - p$

$+3p \quad +3p$

$6 > 4 + 2p$

$-4 \quad -4$

$\frac{2}{2} > \frac{2p}{2}$

$\boxed{1 > p}$

or

$p < 1$

PROBLEM 5: INEQUALITIES WITH SPECIAL SOLUTIONS

Solve each inequality. If the solution is all real numbers, write all real numbers. If there is no solution, write no solution.

a) $9 + 4n < 2(2n - 3)$

$9 + 4n < 4n - 6$

$-4n \quad -4n$

$9 \not< -6$

No Solution

b) $8 + 6x \geq 7x + 2 - x$

$8 + 6x \geq 6x + 2$

$-6x \quad -6x$

$8 \geq 2$ ✓

All real numbers

c) $10 - 8a \geq 2(5 - 4a)$

$10 - 8a \geq 10 - 8a$

$+8a \quad +8a$

$10 \geq 10$ ✓

All real numbers

d) $6m - 5 > 7m + 7 - m$

$6m - 5 > 6m + 7$

$-6m \quad -6m$

$-5 \not> 7$

No solution

$\{\emptyset\}$

e) $2(n - 8) < 16 + 2n$

$2n - 16 < 16 + 2n$

$-2n \quad -2n$

$-16 < 16$ ✓

All real numbers

$\{\mathbb{R}\}$

f) $6w - 4 \leq 2(3w + 6)$

$6w - 4 \leq 6w + 12$

$-6w \quad -6w$

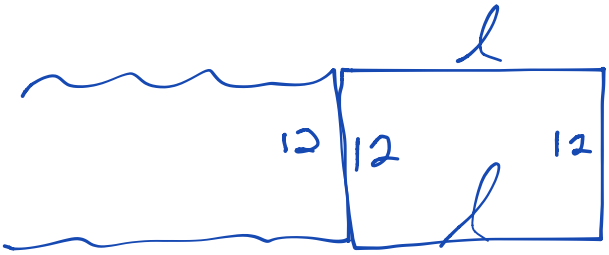
$-4 \leq 12$ ✓

All real numbers

$\{\mathbb{R}\}$

PROBLEM 2: WRITING AND SOLVING A MULTI-STEP INEQUALITY

a) In a community garden, you want to fence in a vegetable garden that is adjacent to your friend's garden that is 12 feet wide. You have at most 42 feet of fence. What are the possible lengths of your garden?



$P = 2l + 2w$

$2l + 2(12) \leq 42$

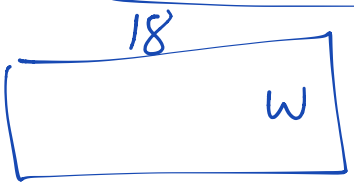
$2l + 24 \leq 42$

$-24 \quad -24$

$\frac{2l}{2} \leq \frac{18}{2}$ $\quad l \leq 9$

At most 9 ft

b) You want to make a rectangular banner that is 18 ft long. You have no more than 48 ft of trim for the banner. What are the possible widths of the banner?



$2(18) + 2w \leq 48$

$36 + 2w \leq 48$

$-36 \quad -36$

$\frac{2w}{2} \leq \frac{12}{2}$ $\quad w \leq 6$

No more than 6 ft

c) On a trip from Buffalo, New York to St. Augustine, Florida, a family wants to travel at least 250 mi in the first 5 h of driving. What should their average speed be in order to meet this goal?

t r d

$d = rt$

$\frac{r(5)}{5} \geq \frac{250}{5}$

$r \geq 50$

At least 50 mph

d) Your cell phone plan costs \$39.99 per month plus \$0.15 for each text message you send or receive over 1000 messages. You have at most \$45 to spend on your cell phone bill. What is the maximum number of text messages that you can send or receive next month?

Let t = text messages over 1000

$39.99 + 0.15t \leq 45$

$-39.99 \quad -39.99$

$\frac{.15t}{.15} \leq \frac{5.01}{.15}$

$t \leq 33.4$

1033 texts

e) An isosceles triangle has at least two congruent sides. The perimeter of a certain isosceles triangle is at most 12 in. The length of each of the two congruent sides is 5 in. What are the possible lengths of the remaining side?

f) The student council wants to rent a ballroom for the junior prom. The ballroom's rental rate is $1500 for 3 hours and $125 for each additional half hour. Suppose the student council raises $2125. What are the possible lengths of time they can afford to rent the ballroom?

Let x = additional halfhours

$1500 + 125x \leq 2125$

$-1500 \quad -1500$

$\frac{125x}{125} \leq \frac{625}{125}$

$x \leq 5$

2½ h

At most 5½ h

g) A sales associate in a shoe store earns $325 per week, plus a commission equal to 4% of her sales. This week her goal is to earn at least $475. At least how many dollars worth of shoes must she sell in order to reach her goal?

Let x = sales

$325 + .04x \geq 475$

$-325 \quad -325$

$\frac{.04x}{.04} \geq \frac{150}{.04}$

$x \geq 3750$

At least #3750

h) The elevator of a building can safely carry no more than 4000 lb. A worker moves supplies in 50-lb boxes from the loading dock to the fourth floor of the building. The worker weighs 210 lb. The cart he uses weighs 95 lb. What is the greatest number of boxes he can move in one trip?

Let x = boxes

$50x + 210 + 95 \leq 4000$

$50x + 305 \leq 4000$

$-305 \quad -305$

$\frac{50x}{50} \leq \frac{3695}{50}$

$x \leq 73.9$

No more than 73 boxes

The worker needs to deliver 275 boxes. How many trips must he make?

$\frac{275}{73} = 3.76$

4 trips